EWMA control charts for exponential data — workbench for considering transformations and numerical methods

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The inherent simplicity of the exponential distribution allows explicit solutions of the average run length (ARL) integral equation for various control charts (here EWMA charts), cf. to some handy methods in Gan (1998); Gan and Chang (2000); Areepong (2009). On the other hand, the omnipresent Markov chain approximation method works feebly (an example will be given, for illustration). Nonetheless, it was used, for example, in Borror et al. (1999); Liu et al. (2007) and elsewhere. Deploying the link between the exponential and the chi-square distribution with two degrees of freedom allows the usage of results (collocation applied to the ARL integral equation) for EWMA S^2 charts in Knoth (2005), where an R package is available. Finally, Monte Carlo studies can be utilized. Thus, one goal of the talk is comparing all these numerical algorithms and promoting efficient ones.

Having all these methods, we can judge whether transformations of the exponential like in Nelson (1994); Kittlitz, Jr. (1999); McCool and Joyner-Motley (1998); Liu et al. (2007), in particular $X^{1/3.6}$ for achieving a bell-shape density function, which mimics the normal case, or other ones like e^{-X} and $\Phi^{-1}(e^{-X})$ to get (exact) beta (uniform in the in-control case) and normal distribution, respectively, are appropriate convenience wrappers or simply diminish the detection performance. Eventually, some recommendations regarding the usage of these transformations will be given.

Keywords: EWMA charts, Exponential data, ARL calculation, Transformations

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