

Comparison of some linear-rank based control charts for location and scale

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In many practical situations involving process monitoring, the data distribution is unknown, and therefore, a nonparametric control chart is required. In this work we consider some linear rank-based control charts for location and scale, and we compare their performance. Suppose we have available a reference sample $(X_1, X_2, \dots, X_m)$ taken from an in-control and stable process, and let $(Y_1, Y_2, \dots, Y_n)$ be the current arbitrary test sample. Assume that these samples are independent, as well as the observations within each sample, and drawn from two populations with continuous cumulative distribution functions F_X and F_Y , respectively. Let us consider the combined ordered sample of size $N=n+m$ and the corresponding vector of indicator random variables $(Z_1, Z_2, \dots, Z_N)$, where $Z_i=1$ if the i -th random variable in the combined ordered sample is from population Y , and $Z_i=0$ if the i -th random variable in the combined ordered sample is from population X , for $i=1, \dots, N=n+m$. A linear-rank based statistic is defined as a linear function of the indicator variables Z_i , and several nonparametric tests are obtained by choosing different sets of scores in this linear combination. For details see, for instance, [1]. In this paper we consider different sets of scores so that several linear rank statistics, popular in the literature, the corresponding charting statistics and the control limits are obtained. Performance of these charts are examined, both in terms of in-control and out-of-control properties, for data from symmetric non-normal distributions with different tail weights.

Keywords: ARL performance, Linear-rank statistics, Nonparametric control charts.

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References

[1] Gibbons, J. D. and Chakraborti, S. (2021). Nonparametric Statistical Inference, 6th edition, Taylor Francis Group, LLC.